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'Exponential Growth' in the Ebola Outbreak: What does it mean?

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Once more we are hearing about 'exponential growth' – popularly some sort of synonym for 'rapid growth' or 'explosive growth' – but actually a technical term with a quite specific meaning. This time the talk is about the ongoing Ebola outbreak in West Africa, understandably causing increasing disruption (is devastation too strong a word?) in the region, and alarm much further afield.

Exponential growth (for example of money in an account) means that something is growing in such a way that its rate of increase is itself growing in such a manner that the growth rate maintains some proportion to the (ever growing) size of the underlying thing. In fact, money lying passively in an interest bearing account grows exponentially. This is also referred to as the phenomenon of 'compound interest' – the interest earned in one accounting period attracts interest in the next accounting period. Clearly, this doesn't actually necessarily mean it's growing very fast. Even 1 percent per annum, or smaller numbers, define specific exponential growth scenarios.

If the exponential growth is maintained long enough, of course, the thing in question (Ebola cases, or bank balance) will eventually double in number/size. If the same underlying pattern of exponential growth is maintained, the growing thing will again eventually double - then being four times the original size. The key thing that makes exponential growth interesting to geeks like mathematicians, statisticians, engineers, physicists, etc. is that that each of these doubling times is the same: if it takes one month for cumulative Ebola deaths to double, and the epidemic is in an exponential phase, then it will take another month for them to double once more, if this dynamic is maintained.

But why would such a thing, this technically defined 'exponential growth' 'be maintained' for periods of time? Is there some principle involved? Does it 'emerge naturally' across a broad range of situations? Does the lens, or perspective, or analytical paradigm or exponential growth help us understand systems by combining analysis with intuition? The answer is largely yes! A brief digression is warranted.

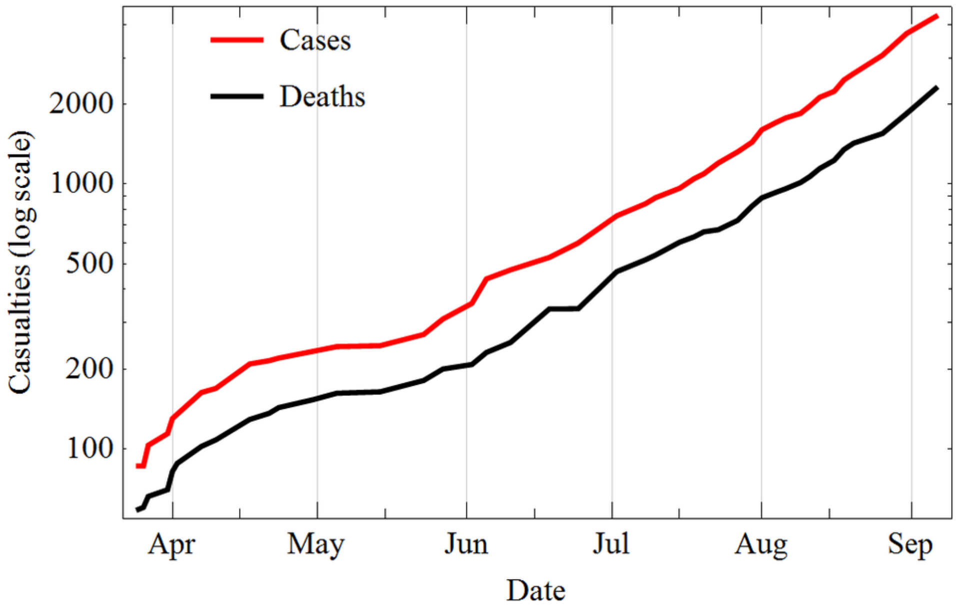
Most people are familiar with the great insight (arguably due to Galileo) which spawned modern physics, that falling objects (in the vicinity of the earth's surface) experience a shared 'acceleration' due to gravity (ignoring air resistance) according to which they pick up, in every second, an extra ten metres per second of *speed* (or '*velocity*' as the purists would say).

The next great leap forward was when Newton explained how this acceleration varies, depending on what planets we are on, how big they are, etc. Thus was modern quantitative natural science born. Almost as fundamentally, ideas around 'exponential growth' are central to pretty much all of applied mathematics, such as physics and engineering, and for many years, they have also attracted a great deal of attention in social dynamics, ecology, and biology, as exponential growth of populations (of mammals, insects, cells, viruses, plants, etc) does indeed emerge as naturally as constant acceleration results from gravitational fields.

Just like the gravity story, the exponential growth story has all sorts of complexities and nuances, which limit its simplistic application. One needs a compass, and some technical training, to wade confidently through all this. Consider the controversial debate around the observation by Thomas Robert Malthus, in 1798, that human populations appear, for periods of time, to grow exponentially. The term 'Malthusian growth' has been redundantly invented to label this phenomenon, and has spawned a polarised debate about whether it implies social challenges for long term coexistence within finite resources, or whether it's the death knell for principles and the starting gun in an amoral contest for survival. Malthus's technical observation is important, and much embellished and used by demographers to provide summaries of fertility and mortality, which drive population dynamics. In short, if women bear children in a stable pattern - meaning that age dependent fertilities (probabilities of bearing children in a given year) are constant for long enough– and if probabilities of surviving from one birthday to the next (actuarial 'life tables') are also constant in time (though allowed to be different for each age group) – then a population will settle into exponential growth or decline. Yes, decline is possible too – it implies a *halving* of population over a characteristic period.

Many of us learned in high school about 'radioactive decay' and the 'half-life' or radioactive materials – the time it takes for half the available material to undergo a 'decay' reaction. This is all driven by the fact that the frequency with which a sample of material produces decay events is proportional to how much of the stuff we are looking at, and has nothing to do with anything else, like 'how old' the sample is. The individual atoms involved do not become more likely to decay, over time, on account of some aging process – they are ageless, facing each year with the same prospects of making it to the next birthday. Complex creatures, like people, do, alas, indeed age. Therein lies one of the complexities of modelling biological systems with exponential processes. Societies change too – usually too quickly for the idealised features of exponential dynamics to persist for very long.

But what about the Ebola then? The figure of Ebola cases and deaths reproduced here (below) comes from the Wikipedia page on the ongoing outbreak – downloaded on 10 September. The horizontal axis is calendar time, and vertical axis does the counting, on a 'logarithmic', or 'log' scale. Log scale means that a particular distance on the axis does not correspond to a particular *count*, but a particular *multiplicative increase in count* – just like old fashioned slide rules. So, some number of millimetres (depending on how the picture is scaled when rendered) represents a doubling of the count. This in turn means that a straight line – which is approximately what we see for the last few months, implies 'exponential growth'. Hooray! We finally reached the point. A steep gradient is more rapid growth, but even a very slow *linear* climb on a plot of this kind would still technically be 'exponential' growth.



Source: *http://en.wikipedia.org/wiki/Ebola_virus_disease*

The first issue one should always engage with, if doing a serious investigation, is how the data is obtained, what it really means and represents, and how reliable it is. For the present purposes, let us put these questions on the side and assume, for the moment, that they are roughly accurate counts of cases and deaths in the heavily affected countries. Interestingly, if we consistently see/miss some specific fraction (like half) of all cases and deaths, then the slope of this graph, which captures the 'doubling time' remains unchanged, and we would still be able to draw some robust conclusions. The Wikipedia graph is sobering: It really looked like things were levelling off in May, when a new, sustained, and lately possibly escalating, phase of growth emerged. We should not over-interpret this picture due to our lack of knowledge of crucial details about the data, but it is worth interpreting the recently sustained nearly linear phase, which implies a doubling time of about a month.

A key concept of mathematical epidemiology, inevitably somewhat butchered, but at least invoked, in the movie 'contagion', and also somewhat harped upon and stretched beyond its valid uses by some professionals who should know better, is the idea of a 'reproductive number' in an outbreak. This approximately refers to how many additional people are, on average, infected by an infected individual over their course of disease. If this average is below one, the epidemic wanes, if it is above one, it grows. If the reproductive number is about one, the disease persists indefinitely. When an epidemic is relatively small, in the sense that the majority of people are uninfected (possible even if the outbreak is devastating at the societal level) as in the Ebola outbreak, and if the growth is exponential, then we can do some simple back-of-the envelope calculations to estimate the reproductive number. And the answer (taking the data naively at face value)is: *about one and a half*! This indeed has plenty of meaning. It means the outbreak, as terrible as it is, can be turned around with a modest improvement in containment efforts (halving transmissions might be a credible immediate goal). It also confirms/underlies what many have said – that Ebola will not thrive in any infrastructure-rich country into which it might spill, and that it has only persisted and grown in West Africa because the response has been so very ineffectual. Action and support, rather than inward looking panic, are really the only appropriate responses from the wealthier countries.

While on the topic of epidemic growth and reproduction numbers, it seems only fair, in South Africa, to ask what these concepts might teach us about ending the HIV epidemic. This is a natural, and rich line of questioning, but, as scientists like to say when confronted by such questions, it is ‘beyond the present scope’.

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